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And the entire moment when moving with proper angular velocity is

$$\int_{\frac{\pi}{2}}^{z_1} -\frac{8}{9} fv^2 \pi r^3 \tan^3 z_1 \frac{\cos^2 z dz}{\sin^3 z} = \frac{4}{9} \pi f v^2 r^3 \tan^3 z_1 \left(\frac{\cos z_1}{\sin^2 z_1} + \log \tan \frac{1}{2} z_1 \right).$$

Hence, $\frac{\frac{4}{9} fv^2 r \tan z_1 [(\cos z_1 \div \sin^2 z_1) + \log \tan \frac{1}{2} z_1]}{(\cos z_1 \div \sin^2 z_1) - \log \tan \frac{1}{2} z_1}$ = a maximum for z_1 , (2)

will give the position of wings when the work being done is a maximum.

Therefore, placing the first differential coefficient = 0, and solving for $\log \tan \frac{1}{2} z_1$, there results

$$\log \tan \frac{1}{2} z_1 = (\cos z_1 \div \sin^2 z_1) [1 + \cos^2 z_1 \pm \sqrt{(4 + \cos^4 z_1)}].$$

Substituting in (2) this second value, the only one applicable,

$$\frac{\tan z_1 [1 + 1 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{1 - 1 - \cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}} = \frac{\tan z_1 [2 + \cos^2 z_1 - \sqrt{(4 + \cos^4 z_1)}]}{\sqrt{(4 + \cos^4 z_1)} - \cos^2 z_1}$$

$$= -\tan z_1 + \frac{1}{2} \tan z_1 [\cos^2 z_1 + \sqrt{(4 + \cos^4 z_1)}] = \text{maximum for } z_1.$$

Differentiating a second time and reducing,

$$\cos^6 z_1 - \frac{7}{2} \cos^4 z_1 - 2 \cos^2 z_1 + 2 = 0, \text{ from which}$$

$$\cos^2 z_1 = 0.55154; \therefore z_1 = 42^\circ 2'.$$

SOLUTION OF A PROBLEM.

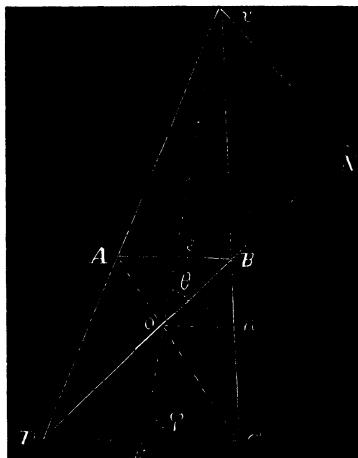
BY PROF. C. M. WOODWARD, WASHINGTON UNIV., ST. LOUIS, MO.

Problem.—Given a frustum of an oblique cone with a circular base; the frustum is cut in two by a plane perpendicular to the principal plane of the cone, and tangent to the two bases. Find the ratio of the volumes of the two parts of the frustum.

Solution.—Let $vABCD$ be the section of the cone made by the principal plane, and let DBN be the trace of the intersecting plane perpendicular to it. The section, projected in DB , is obviously an ellipse of which DB is the major axis. Let $ov = h$, $As = r$, $Dk = R$ and $on = x$. Then

$$DB = (R+r) \frac{\sin \varphi}{\sin \theta}.$$

The minor axis is the chord of a circle whose radius is $\frac{1}{2}(R+r)$, and whose dist. from the center is $\frac{1}{2}(R-r)$; its length is therefore $2\sqrt{(Rr)}$.



The volume of the cone $v-DB$ is

$$V_0 = \pi \frac{R+r}{2} \cdot \frac{\sin \varphi}{\sin \theta} (Rr)^{\frac{1}{2}} \frac{h \sin \theta}{3} = \frac{1}{3} \pi h \sin \varphi \frac{R+r}{2} (Rr)^{\frac{1}{2}},$$

but $\frac{R+r}{2} = \frac{Rr}{x}$; hence $V_0 = \frac{1}{3x} \pi h \sin \varphi R^{\frac{3}{2}} r^{\frac{3}{2}}$.

Now $vs = \frac{hr}{x}$ and $vk = \frac{hR}{x}$, hence

cone $v-AB = V_1 = \frac{1}{3x} \pi h \sin \varphi r^3$, cone $v-CD = V_2 = \frac{1}{3x} \pi h \sin \varphi R^3$.

$$\text{Vol. } ABD = V_0 - V_1 = \frac{1}{3x} \pi h \sin \varphi (R^{\frac{3}{2}} - r^{\frac{3}{2}}) r^{\frac{3}{2}},$$

$$\text{Vol. } BDC = V_2 - V_0 = \frac{1}{3x} \pi h \sin \varphi (R^{\frac{3}{2}} - r^{\frac{3}{2}}) R^{\frac{3}{2}},$$

hence $\frac{\text{Vol. } ABD}{\text{Vol. } BDC} = \frac{\sqrt{r^3}}{\sqrt{R^3}}$.

From the volumes of the three cones we see that the elliptical cone is a mean proportional between the other two. (This striking analogy between the cones $v-AB$, $v-BD$, $v-CD$, and the triangles AvB , DvB , DvC , I had never before noticed, though it must have been well known. Neither had I ever observed that the semi-conjugate axis of such a conic section is a mean proportional between the radii of the bases of the frustum.)

It is obvious that the plane AC divides the frustum in the same ratio as the plane BD .

If the altitude of the frustum is p we have $h \sin \varphi : x = p : (R-r)$ and

$$V_0 = \frac{1}{3} \pi p [V(R^3 r^3) : (R-r)],$$

$$V_1 = \frac{1}{3} \pi p [r^3 : (R-r)], \quad V_2 = \frac{1}{3} \pi p [R^3 : (R-r)],$$

$$V_0 - V_1 = \frac{1}{3} \pi p \frac{\sqrt{R^3} - \sqrt{r^3}}{R-r} r^{\frac{3}{2}},$$

$$V_2 - V_0 = \frac{1}{3} \pi p \frac{\sqrt{R^3} - \sqrt{r^3}}{R-r} R^{\frac{3}{2}}.$$

SOLUTION OF PROBLEM 255.

BY PROF. W. W. HENDRICKSON, NAVAL ACADEMY, ANNAPOLIS, MD.

DENOTING the distance AB by a , and taking the axes as represented in the figure, the equations to the lines AC and BC are (1.) $y = x \tan(\varphi + a)$,